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Robust data reduction of high spatial resolution optical vibration measurements

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Abstract

In several mechanical engineering applications high spatial resolution measurements are required. Adapted optical measurement instruments such as the laser scanning Doppler vibrometer (SLDV) and the electronic speckle pattern interferometer (ESPI) exist to perform this task. The result of this high spatial resolution measurement is that a large amount of data is available which has to be processed. In addition, care should be taken when processing the measurements, because locations with poor measurement quality typically exist. In this article a method will be developed to reduce the amount of measurements so that further processing is less time consuming. In addition, as a side effect of the data reduction method, poor quality measurements will be filtered and the overall SNR will improve. The method uses an iterative robust two-step spline approximation with an automatic model order determination procedure. The validation of the technique is performed both on scanning laser vibrometer measurements of a car door and a circuit board and on ESPI data.

1. Overview of existing techniques

The use of spatially dense optical vibration measurement techniques, such as the scanning laser Doppler vibrometer (SLDV) [1] or the electronic speckle pattern interferometer (ESPI) [2] results in a large amount of data which has to be stored. Because further processing (e.g., extracting modal parameters) of this large amount of data is computationally very expensive, data reduction procedures should be used. In addition, using a data reduction procedure the optical vibration measurements (which are often quite noisy, with outliers at several locations) can be smoothed.

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The data reduction procedure transforms the complete set of measurements to a reduced set of 'virtual' measurements which contains approximately the same amount of information. An important assumption of the data reduction method is that the transform should be linear. This is necessary to guarantee that the poles of the mechanical system under investigation are invariant under the transformation (this eliminates the use of all lossless image coding techniques which are available in literature [3]).

Assume that the high spatial resolution frequency response function (FRF) measurements are written in matrix form $\mathbf{H}$, with $\mathbf{H}$ an N_o by N_f matrix, where N_o is the number of measured output locations and N_f the number of measured frequencies (with usually $N_o \gg N_f$). The data reduction process consists of the following transformation: $\mathbf{TH} = \mathbf{H}_{vo}$, with $\mathbf{T}$ the N_{vo} by N_o transformation matrix and $\mathbf{H}_{vo}$ the reduced virtual FRFs, where N_{vo} are the number of virtual outputs ($N_{vo} \ll N_o$).

One possibility to perform the transformation is to use the singular value decomposition (SVD) of $\mathbf{H}$. Although the SVD based technique was applied with success in literature for moderate size data sets (first in Ref. [4] and later in Ref. [5]) the computational load to perform the SVD is too expensive (order $N_o N_f^2$ calculations) when several hundreds of thousands of outputs are measured (in the case of ESPI measurements at one million locations are often available). Also, because the SVD is a least-squares procedure, the presence of outliers in the measurements can lead to an incorrect reduced data set.

A computationally much more attractive and more robust alternative was proposed by Arruda et al. [6,7]. Their so-called regressive discrete Fourier series (RDFS) is a data reduction technique which uses complex exponential basis functions to approximate the vibration patterns at the N_f different measurement frequencies. The coefficients of the RDFS approximation of a vibration pattern at frequency i (for $i = 1, \ldots, N_f$) are considered as the reduced virtual measurements (i.e., columns in $\mathbf{H}_{vo}$). When the measurement locations are positioned on a square grid (which is true for ESPI measurements where the measurements locations are pixels of an image) the order $N_o \sqrt{N_{vo}} N_v$ computations are needed. Examples in literature have shown that the RDFS method works well. However, some improvements can be made to make it faster, more generally applicable, robust and less user interactive.

In this article, a method also belonging to the class of regressive techniques (such as the RDFS) will be developed. In particular, in contrast to existing techniques it has the following advantages:

(1) The use of a spline basis instead of a Fourier basis gives rise to sparse banded matrices resulting in a smaller computational load (see Section 2.1).
(2) An iterative approach is used to allow more general measurement grids (the RDFS is limited to rectangular grids) (Section 2.2).
(3) An algorithm is developed (in Section 2.3) to automatically determine the correct model order (i.e., N_{vo}).
(4) A two-stage scheme is used to make the data reduction method more robust to outliers (Section 2.4).

The method, which is discussed in detail in Section 2, is illustrated on SLDV measurements of a circuit board, a car door and a brake drum in Sections 3.1–3.3. Finally, conclusions will be drawn in Section 4.

2. The proposed method

2.1. The spline approximation method

The proposed data reduction method is based on the use of a spline approximation $s(x_i, y_j)$ of a complex valued vibration pattern $z(x_i, y_j)$, with $i = 1, \ldots, N$ and $j = 1, \ldots, M$, where $NM = N_o$ (i.e., the outputs are positioned on a rectangular $N \times M$ grid). The spline $s(x_i, y_j)$ on the interval $[x_1, x_N] \times [y_1, y_M]$ can be expressed as [8]:

$$s(x, y) = \sum_{i=-k}^{g} \sum_{j=-l}^{h} c_{i,j} N_{i,k+1}(x) M_{j,l+1}(y), \tag{1}$$

where $N_{i,k+1}$ and $M_{j,l+1}$ are B-splines with knot sequences $\lambda_0, \ldots, \lambda_{g+1}$ and $\mu_0, \ldots, \mu_{h+1}$ respectively (note that $\lambda_0 = x_1$, $\lambda_{g+1} = x_N$, $\mu_0 = y_1$ and $\mu_{h+1} = y_M$). k and l are the degrees of the spline in the x and y directions, respectively. The spline coefficients $c_{i,j}$ in Eq. (1) are obtained using the following least-squares optimization problem (with $||.||$ the Frobenius norm):

$$\min_{\mathbf{c}} ||\mathbf{Z} - \mathbf{McN}^{\mathrm{T}}||, \tag{2}$$

where

$$\mathbf{Z} = \begin{pmatrix} z(x_1, y_1) & \ldots & z(x_N, y_1) \\ \vdots & \vdots & \vdots \\ z(x_1, y_M) & \ldots & z(x_N, y_M) \end{pmatrix}$$

is the vibration shape, and

$$\mathbf{c} = \begin{pmatrix} c_{-k,-l} & \ldots & c_{g,-l} \\ \vdots & \vdots & \vdots \\ c_{-k,h} & \ldots & c_{g,h} \end{pmatrix} \tag{3}$$

the spline coefficients. Further,

$$\mathbf{N} = \begin{pmatrix} N_{-k,k+1}(x_1) & \ldots & N_{g,k+1}(x_1) \\ \vdots & \vdots & \vdots \\ N_{-k,k+1}(x_N) & \ldots & N_{g,k+1}(x_N) \end{pmatrix}, \tag{4}$$

$$\mathbf{M} = \begin{pmatrix} M_{-l,l+1}(y_1) & \ldots & M_{h,l+1}(y_1) \\ \vdots & \vdots & \vdots \\ M_{-l,l+1}(y_M) & \ldots & M_{h,l+1}(y_M). \end{pmatrix} \tag{5}$$

are the B-spline base functions evaluated at the x and y grid points. Using elementary matrix operations it can be shown that the solution of Eq. (2) is given by the following expression:

$$\mathbf{c} = (\mathbf{M}^{\mathrm{T}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{ZN}(\mathbf{N}^{\mathrm{T}}\mathbf{N})^{-1}. \tag{6}$$

For all measured frequencies $f_1, \ldots, f_N$ the output locations and the spline knots are the same, and therefore the matrices **N** and **M** are independent of the frequency and should be calculated only once. In addition, the matrices **N** and **M** are sparse matrices with a band structure (the columns in $\mathbf{N}$ and $\mathbf{M}$ have at most $N(k+1)/(g+1)$ and $M(l+1)/(h+1)$ non-zero elements, respectively). This implies a large reduction in the number of operations needed to perform the products $(\mathbf{M}^T\mathbf{M})^{-1}$, $(\mathbf{N}^T\mathbf{N})^{-1}$ and $\mathbf{M}^T\mathbf{Z}\mathbf{N}$ in Eq. (6).

The complete data reduction procedure, starting from the full FRF matrix $\mathbf{H}$ to compute the reduced data set $\mathbf{H}_{vo}$, is given in the following algorithm:

Algorithm 1. Spline data reduction

(1) Select spline degrees k and l (for simplicity the degree in x and y direction will be taken equal from now on, i.e., $k = l$).
(2) Select number of spline knots g and h (the knots are spaced equidistantly, and $g = h$ without loss of generality). From now on the spline model order is written as $N_s = g + k + 1$ (i.e., this is equal to the number of columns in $\mathbf{N}$ and $\mathbf{M}$).
(3) Compute $\mathbf{P} = (\mathbf{M}^T\mathbf{M})^{-1}$ and $\mathbf{Q} = (\mathbf{N}^T\mathbf{N})^{-1}$ (this requires order $N_sN((k+1)/(g+1)) + N_s^2$ operations).
(4) FOR $i = 1, \ldots, N_f$
    (a) Compute $\mathbf{R} = \mathbf{M}^T\mathbf{Z}\mathbf{N}$, with $\mathbf{Z}$ the vibration pattern at frequency f_i, i.e., $\mathbf{Z}$ is one column from the FRF matrix $\mathbf{H}$: $\text{vec}(\mathbf{Z}) = \mathbf{H}(:, i)$.
    $(N_sMN((k+1)/(g+1)) + N_s^2N((k+1)/(g+1))$ operations, where the first term dominates since $N_s \ll M$).
    (b) Solve Eq. (6), by calculation the matrix product $\mathbf{c} = \mathbf{PRQ}$
    (N_s^3 operations).
    (c) Store the N_s^2 spline coefficients $\mathbf{c}$ as one column of the reduced FRF matrix: $\mathbf{H}_{vo}(:, i) = \text{vec}(\mathbf{c})$.
(5) END

The spline data reduction algorithm requires order $N_fMNN_s((k+1)/(g+1)) = N_fN_o\sqrt{N_{vo}}((k+1)/(g+1))$ operations. This is a factor $(g+1)/(k+1)$ smaller than for the RDFS method (typically, if $k = 2$ and $g = 14$ the spline method is five times faster than the RDFS).

2.2. The missing value problem

In Section 2.1 it was assumed that the output locations were positioned on a rectangular grid. Although this is quite common for many applications on panel-like structures, it is generally too restrictive. In the algorithm below, the spline data reduction method is generalized to measurement grids where particular areas in a rectangular area do not contain measurement locations (note that it is still assumed that the output locations are positioned on a rectangular grid, although missing values may occur):

Algorithm 2. Missing value spline data reduction
FOR $i = 1, \ldots, N_f$

(1) Let the matrix $\mathbf{Z}$ be the rectangular vibration pattern at frequency f_i, where the locations $z(x_{I_1}, y_{I_1}), \ldots, z(x_{I_n}, y_{I_n})$ with indices in I are missing (for instance locations at the border or

holes in a structure). Denote J the set of matrix indices which are measured and $K = \{1, \ldots, N_o\} = I \cup J$.

(2) Obtain starting values for the missing values: for each closed region in $(x_{I_1}, y_{I_1}), \ldots, (x_{I_n}, y_{I_n})$ interpolate the missing values with a cubic polynomial $p(x, y)$. This polynomial is calculated as the approximating polynomial of the measurements at the border values of the closed region. Substitute the polynomial values for the missing values: i.e., put $z(x_{I_1}, y_{I_1}) = p(x_{I_1}, y_{I_1}), \ldots, z(x_{I_n}, y_{I_n}) = p(x_{I_n}, y_{I_n})$.

(3) FOR $i = 1, \ldots, N_{iter}$

   (a) Compute the spline coefficients $\mathbf{c}$ in Eq. (6) from the matrix $\mathbf{Z}$ (including the estimations of the missing values). The same procedure as in Algorithm 1 is used.

   (b) Calculate the spline fit $\mathbf{S} = \mathbf{M}^{\mathrm{T}}\mathbf{c}\mathbf{N}$.

   (c) Put $z(x_{I_1}, y_{I_1}) = s(x_{I_1}, y_{I_1}), \ldots, s(x_{I_1}, y_{I_1}) = s(x_{I_n}, y_{I_n})$ as new values for the missing data.

(4) END

It can be proved that the iterative method converges to the least-squares solution (where the missing values are not used). Compared to the rectangular grid method in Section 2.1, the missing value spline data reduction method requires N_{iter} times more operations (with N_{iter} typically 10).

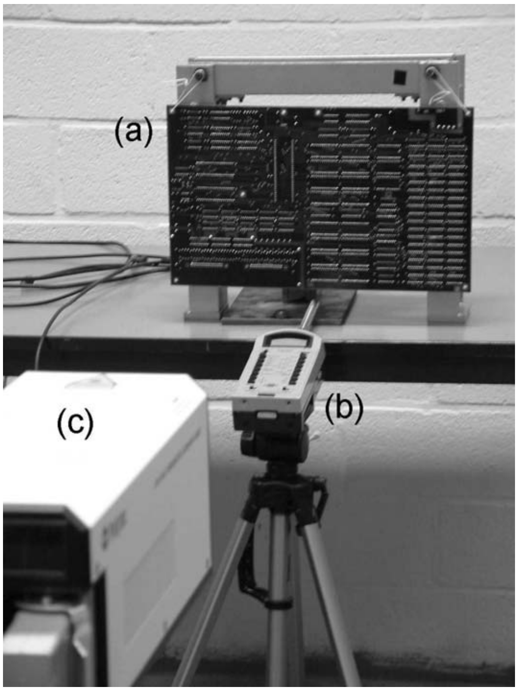


Fig. 1. Measurement set-up of the circuit board measurement: (a) circuit board, (b) sonometer with microphone, and (c) laser Doppler vibrometer.

2.3. The model determination method

A key issue in the success of the data reduction method is the selection of the proper spline model order N_s (note that $N_s = g + k + 1$ with g the number of internal spline knots and k the degree of the spline). Indeed, if N_s is taken too low, the fit of the vibration shape will be poor (note that in that case the system poles will be still correct but the mode shapes will be under fitted). On the other hand when N_s is too large, the measurement noise will be fitted and the reduced data are not smooth enough.

In this section a tool will be developed to identify the proper model order. The tool aims at quantifying whether the residuals of a spline (i.e., $s - z$) with a certain model order are spatially uncorrelated (in that case the residuals contain the measurement noise while the spline fit models the true underlying vibration shape). A test statistic T based on the number of sign changes in the residuals will be used thereto:

Algorithm 3. Model order determination

(1) Use the vibration shape $\mathbf{Z}$ at frequency N_f.
(2) FOR $N_s = 3, \ldots, N$
    (a) Compute the model order N_s spline fit $\mathbf{S}$ of $\mathbf{Z}$ as in Sections 2.2 or 2.3.

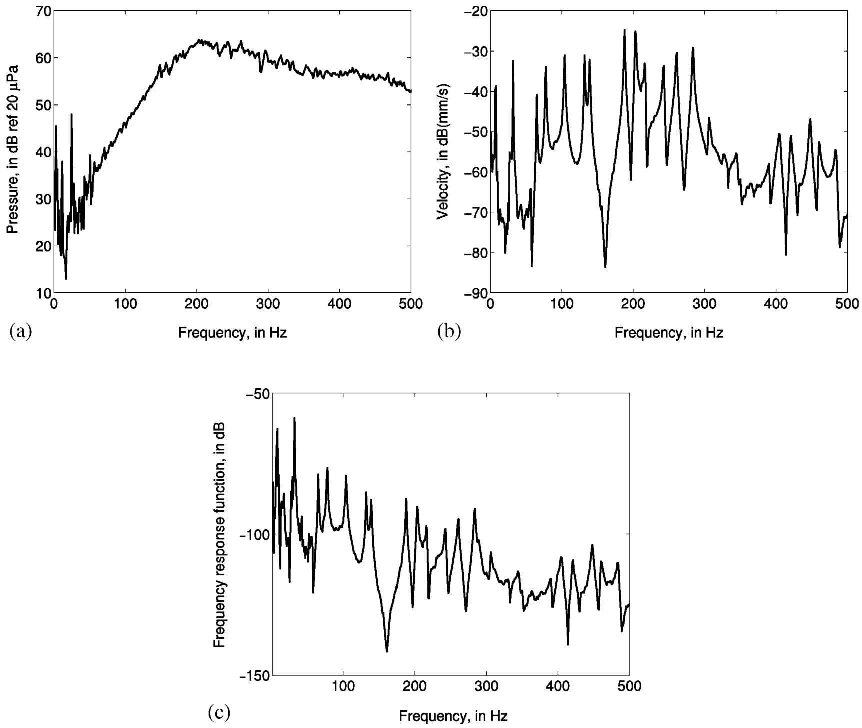


Fig. 2. Spectra of the circuit board measurement: (a) acoustic pressure, (b) velocity, and (c) FRF: velocity over pressure.

(b) Calculate the residuals **Q** of the spline fit: $\mathbf{Q} = \mathbf{S} - \mathbf{Z}$.
(c) Evaluate the number of sign changes U in **Q**: $U = \sum_{i=1}^{N_o-1} \mathrm{sgn}(q(x_{I_i}, y_{I_i})q(x_{I_{i+1}}, y_{I_{i+1}}))$, where $\mathrm{sgn}(x) = 0$ if $x \geqslant 0$ and $\mathrm{sgn}(x) = 1$ if $x < 0$.
(d) Compute the test statistic T: $T = \frac{U - N_o/2}{\sqrt{N_o/4}}$. When the residuals are uncorrelated, the test statistic T has a standard normal distribution (see Ref. [9]).

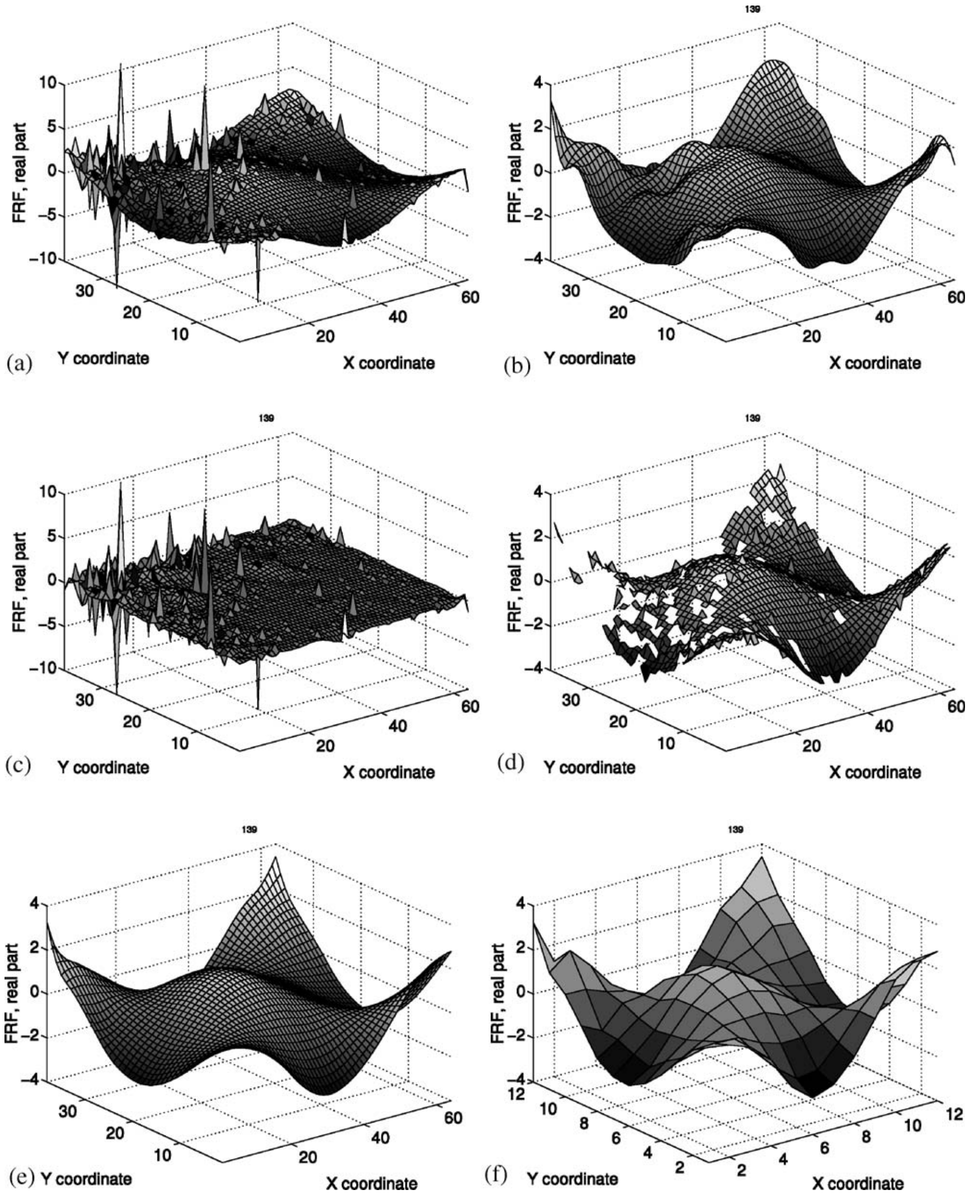


Fig. 3. Raw and processed operating deflection shapes of the circuit board at 139 Hz: (a) raw operating deflection shape, (b) spline fit of (a), (c) residuals of spline fit: (a)–(b), (d) raw measurement with outliers removed, (e) iterative spline fit of (d), (f) coefficients of the fit in (e).

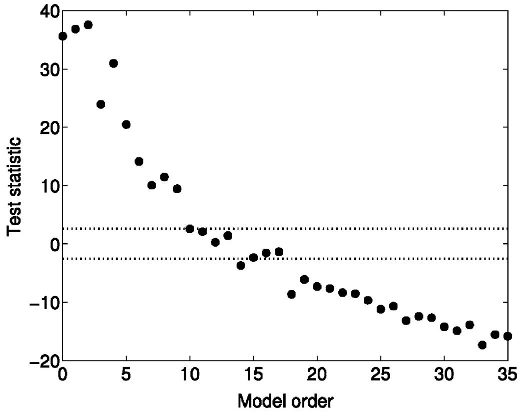


Fig. 4. Test statistic of the spline fit of the circuit board measurement at 471 Hz versus the increasing model order (i.e., number of spline knots plus three).

(3) The correct model order N_s is the smallest order for which T falls into the $\alpha\%$ (e.g., $\alpha = 5$) confidence level of the standard normal distribution (or the order with the smallest $|T|$ if the latter does not occur).

2.4. Robustification of the method

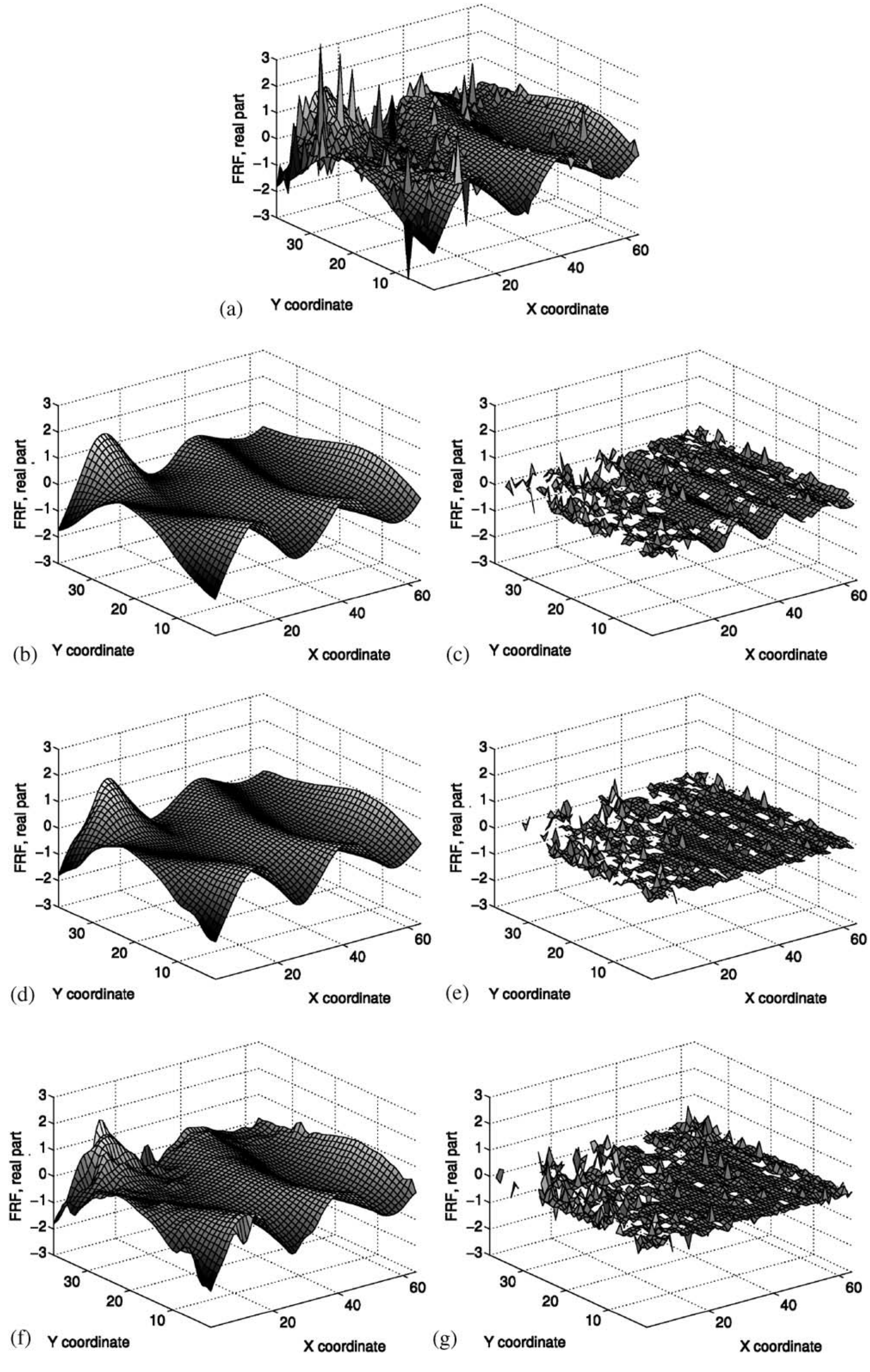
Since the spline data reduction method is a least-squares procedure, it will give poor results when outliers are present in the measurements. In this section an adaptation of the method will be proposed that can handle a small amount (typically up to 10%) of outliers. This is essential when processing optical measurements, which typically contain several very poor quality measurement locations (this is in particular true for SLDV and ESPI measurements on untreated surfaces). The proposed robust method is a two-stage approach: first the outliers are detected from the residuals of the least-squares spline fit, then a second least-squares fit of the operating shape with the outliers removed is performed.

In more detail the algorithm is as follows:

Algorithm 4. Robust spline data reduction
FOR $i = 1, \ldots, N_f$

(1) Compute the least-squares spline coefficients $\mathbf{c}$ in Eq. (6) from the vibration pattern $\mathbf{Z}$ at frequency f_i and the spline fit $\mathbf{S} = \mathbf{M}^{\mathrm{T}}\mathbf{cN}$
(2) Calculate the spline fit residuals: $\mathbf{Q} = \mathbf{S} - \mathbf{Z}$.
(3) Evaluate the MAD [10] of the spline fit residual as a robust measure of the standard deviation: $MAD(\mathbf{Q}) = median(|\mathbf{Q} - median(\mathbf{Q})|)$.

Fig. 5. Raw and processed operating deflection shapes of the circuit board at 471 Hz: (a) raw measurement, (b) iterative spline fit with model order 6, (c) residuals of the iterative spline fit with model order 6, (d) iterative spline fit with model order 10, (e) residuals of the iterative spline fit with model order 10, (f) iterative spline fit with model order 20, (g) residuals of the iterative spline fit with model order 20.


(a)
FRF, real part
Y coordinate
X coordinate
(b)
FRF, real part
Y coordinate
X coordinate
(c)
FRF, real part
Y coordinate
X coordinate
(d)
FRF, real part
Y coordinate
X coordinate
(e)
FRF, real part
Y coordinate
X coordinate
(f)
FRF, real part
Y coordinate
X coordinate
(g)
FRF, real part
Y coordinate
X coordinate

(4) Find all outlier output locations (x_i, y_i) where $q(x_i, y_i) > \frac{1.96 MAD(\mathbf{Q})}{0.6745}$ (the factor 1.96 gives the 5% confidence level, while the denominator term 0.6745 is used to make the MAD consistent with the standard deviation [10]). Label these locations $(x_i, y_i), \ldots, (x_m, y_m)$ as missing values (note that at most 10% outliers are taken, thus $m < 0.1 N_o$).

(5) Compute the least-squares spline coefficients and spline fit of the vibration pattern **Z** with the missing values at the outliers outputs (using the procedure in Section 2.2).

END

3. Experimental results

In the following sections the robust, missing value spline data reduction method with automatic model order determination is validated for two experimental examples: a circuit board (Section 3.1) and a car door (Section 3.2).

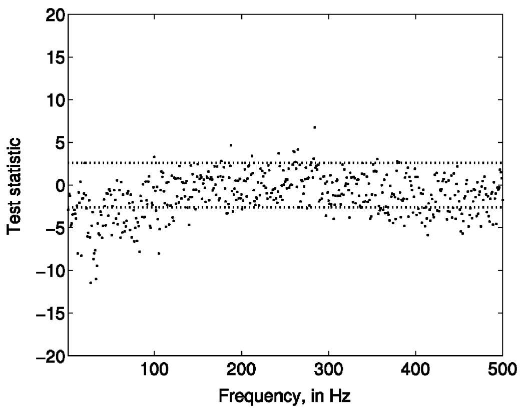


Fig. 6. Test statistic of the spline fit of the circuit board measurement with a fixed model order 10 as a function of the frequency.

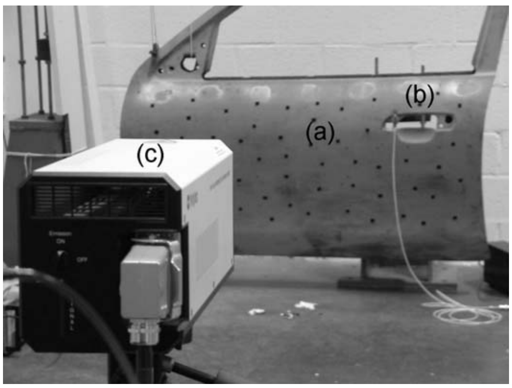


Fig. 7. Measurement set-up of the car door measurement: (a) car door, (b) shaker position with a force cell behind the door, and (c) laser Doppler vibrometer.

3.1. Circuit board data

The set-up of the circuit board measurement is shown in Fig. 1. A loudspeaker is used to excite the circuit board (in the middle, behind the circuit board in Fig. 1). A B&K sound level meter is used to measure the acoustic pressures, while a Polytec scanning laser Doppler vibrometer is used to measure the velocity at about 2400 locations (the measurements are performed up to 500 Hz with 1 Hz frequency resolution). From the measurement spectra in Fig. 2 it can be seen that only the frequencies above about 150 Hz were excited properly; below 150 Hz the FRFs were to noisy to allow a correct identification of the modes (this is due to the limited size of the loudspeaker).

In Fig. 3 the robust spline fit of one single vibration pattern (at 139 Hz) is illustrated. Fig. 3b shows the least-squares spline fit which is computed from the raw measurement. It is clear that the outliers in the measurement distort the spline fit (due to the outliers the least-squares fit becomes less smooth). The residuals of the least-squares fit are shown in Fig. 3c. From these residuals the outliers are identified and these locations are labelled as missing values. The remaining (good quality) locations are shown in Fig. 3d. From the surface in Fig. 3d the robust spline is computed (see Fig. 3e). It can be seen that the robust spline fit (Fig. 3e) is much smoother than the least-squares fit (Fig. 3b). In contrast to the raw data set which contained 2400 output locations, only

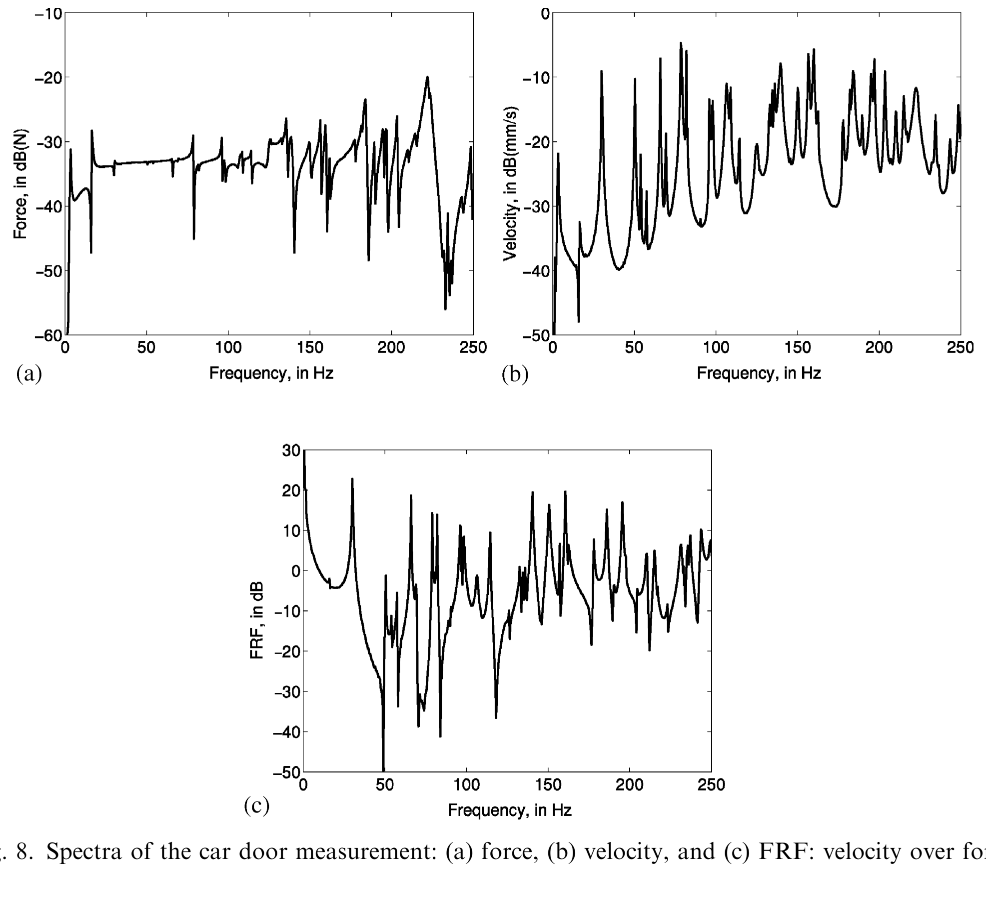


Fig. 8. Spectra of the car door measurement: (a) force, (b) velocity, and (c) FRF: velocity over force.

144 reduced virtual outputs are required when using the spline coefficients (which are shown in Fig. 3f). In addition, the reduced measurement is much less noisy than the raw data.

The results of the model order determination procedure on the circuit board data at frequency 471 Hz are shown in Fig. 4: the test statistic T indicates that all model order between 10 and 17 (except 14) give rise to uncorrelated residuals. Indeed, when looking at Fig. 5 it is clear that model order 10 should be taken instead of, for example, 6 or 20: for model order 6 the residuals (Fig. 5b) still contain information of the vibration shape, while for model order 20 the spline (given in Fig. 5f) starts to fit the noise of in the raw measurement (which is shown in Fig. 5a).

For the spline data reduction method the model order should be equal for all frequencies. In order to see if the chosen model order (i.e., 10) is adequate for all frequencies, the test statistic is evaluated during the data reduction procedure as shown in Fig. 6. From this figure it is clear that model order 10 gives good results (i.e., uncorrelated spline fit residuals) for most frequencies (except for the frequency region below 100 Hz, where the noise level is unacceptably high).

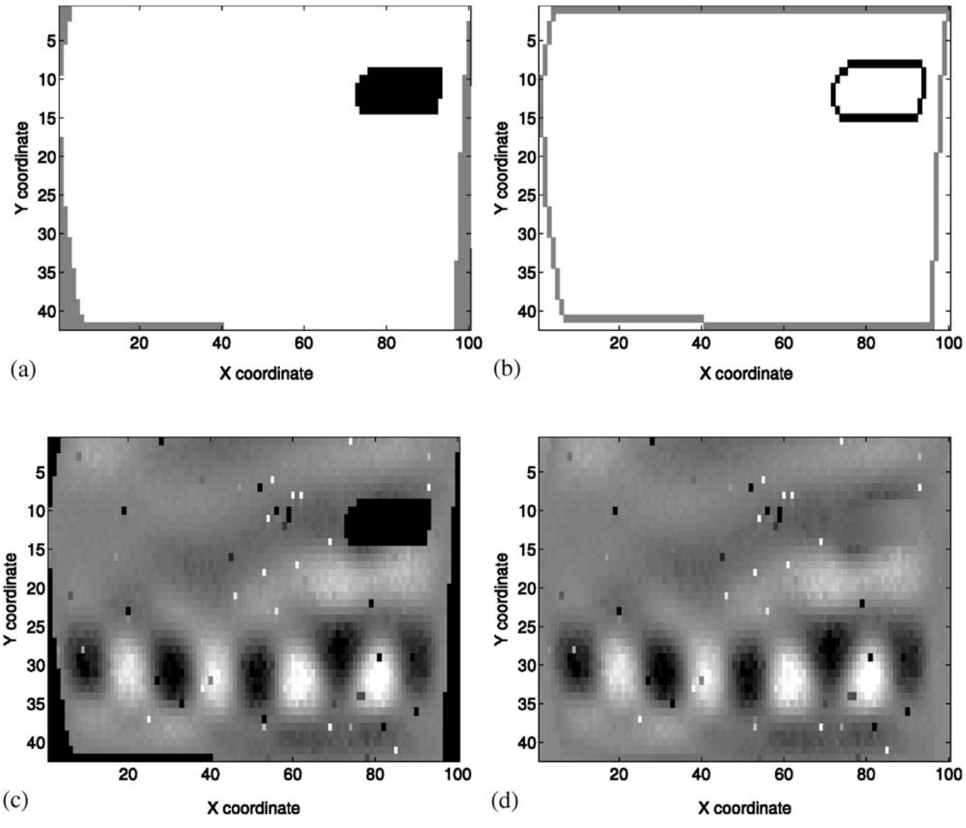


Fig. 9. Estimation of starting values for missing values. (a) Different regions in the rectangular measurement area: white = object, grey = border, black = hole. (b) Border of the different regions in (a). (c) Example measurement (at 235.5 Hz) with missing values indicated in black. (d) Example measurement from (c), missing values are obtained by extrapolation of the border values.

The circuit board measurement is an example where the output locations were positioned on a rectangular grid. In the next section the use of the missing value algorithm (see Section 2.2) will be demonstrated on a car door.

3.2. Car door data

The car door was excited with an electromagnetical shaker, on which a force cell was mounted. The velocities up to 250 Hz were measured at 3859 locations with the scanning laser vibrometer (see the measurement set-up in Fig. 7). The FRFs in Fig. 8 show that about 40 modes are present in the selected frequency band. Moreover, from the same figure it can also be seen that the quality of the measurements is quite good (about 40 dB). This was the case because the door was treated to retro-reflective paint. In order to artificially introduce outliers in the measurement, black velvet spots were glued to the car door surface (see the black spots in Fig. 7). A reference measurement was also performed prior to gluing in order to be able to compare the results of the processed raw (noisy) data (the excitation amplitude was also increased with 20 dB to improve the SNR).

As a first step in the missing value spline data reduction procedure in Section 2.2 starting values were computed for the missing values (i.e., the locations where no measurement is available are shown in black and grey in Fig. 9a). Hereto, a cubic polynomial approximation of the measurement values at the corresponding borders (black and grey in Fig. 9b) is calculated.

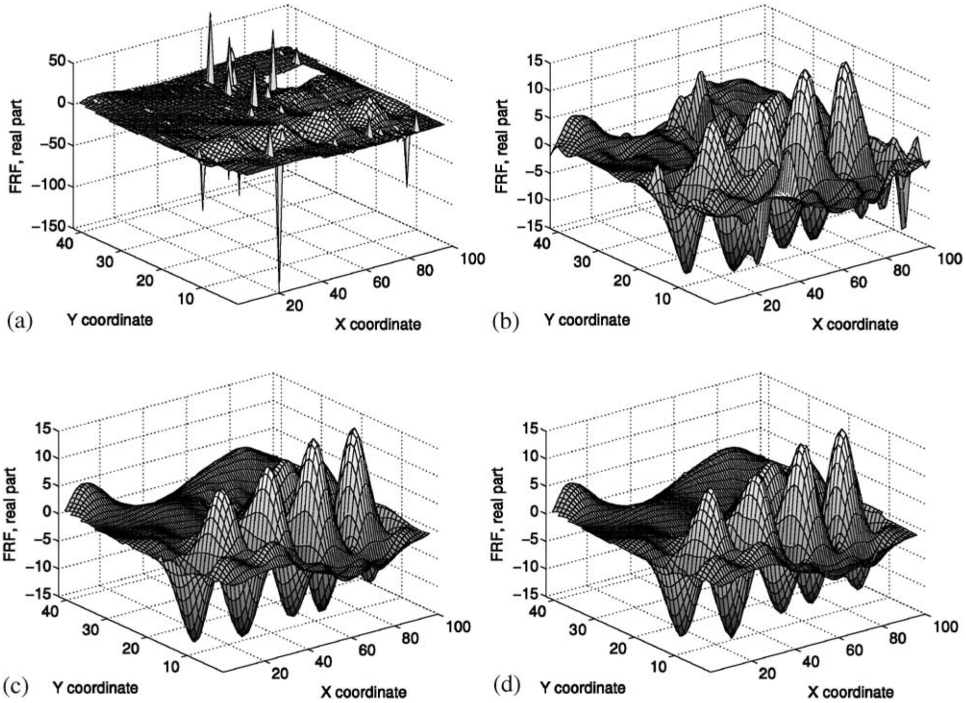


Fig. 10. Iterative spline fit of the car door measurement. (a) Raw operating deflection shape at 235.5 Hz of the measurement. (b) Spline fit of the deflection shape in (a). (c) Iterative spline fit of the deflection shape in (a). (d) Good quality reference measurement (without outliers and SNR is 20 dB larger than SNR of the measurement in (a)).

Table 1
Correlation between reference data and raw or processed data for the car door measurements

Input data H	Correlation with H_{ref} (in %)
Raw data	64.3
Spline fit	93.1
Robust fit	99.4

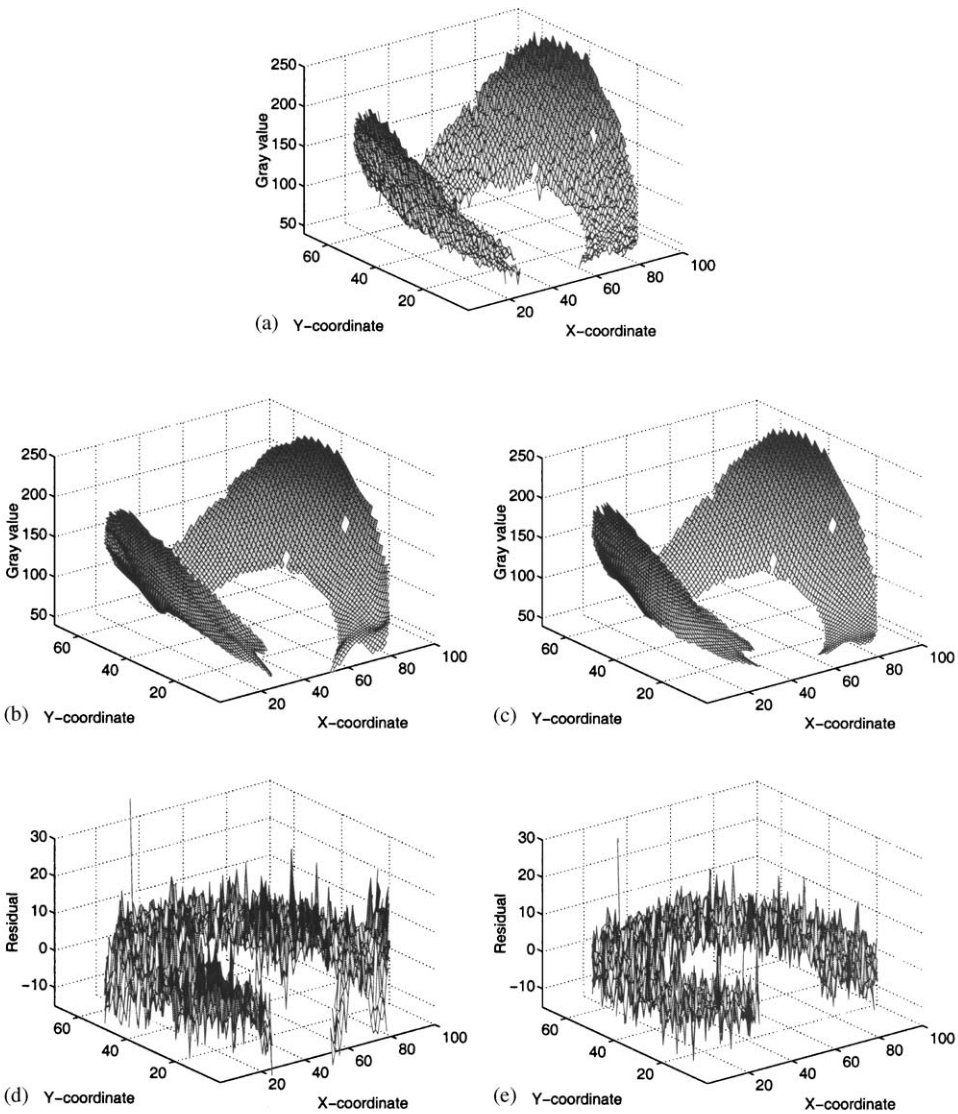


Fig. 11. Iterative spline fit of the ESPI brake drum measurement. (a) Raw operating deflection shape of the measurement. (b) Spline fit of the deflection shape in (a). (c) Iterative spline fit of the deflection shape in (a). (d) Residual of the spline fit. (e) Residual of the iterative spline fit.

Table 2
Test statistics T (see Section 2.3) for the spline fit and the robust spline fit of the brake drum vibration pattern

Input data H	Test statistic T
Spline fit	45.5
Robust fit	−0.9

The function values of the cubic polynomial at the black and grey areas are used as starting values for the missing data values (in Fig. 9c the missing values are given in black, while Fig. 9d represents the polynomial fit starting values).

The result of the missing value spline fit data reduction procedure after 10 iterations is shown in Fig. 10b. When comparing the raw vibration shape (Fig. 10a) with the spline fit (Fig. 10b) it can be seen that the fit is still quite poor near locations with outliers. The robust fit in Fig. 10c gives a much better result. Indeed, when comparing the robust fit (Fig. 10c) with the reference data set (without outliers) one can see that the difference is very small. Quantitatively, the agreement between the reference vibration shape H_{ref} and the raw and processed vibration shapes H can be represented using the correlation coefficient: $r = \mathrm{cov}(H, H_{ref})/\sqrt{\mathrm{var}(H)\,\mathrm{var}(H_{ref})}$. For the robust fit, almost 100% correlation is obtained, while the raw data only gave 64.3% correlation (see Table 1).

3.3. ESPI data

The proposed data reduction technique can also be applied to ESPI measurements. An example of a double pulsed ESPI measurement on a brake drum is given in Fig. 11 (for clarity of the figure the shapes have been subsampled four times). From the residuals in Figs. 11d and e it is clear that the iterative spline (100 iterations) properly fits the vibration shape, while in the classical spline fit some information is still present in the residual. This can be seen quantitatively by virtue of the test statistic T which is given in Table 2 for the brake drum example.

4. Conclusions

In this article a data reduction method based on a robust spline fit has been proposed. Because of the banded structure of the spline base matrices the method is computationally very attractive. As opposed to existing methods (such as the RDFS), the developed method can also handle missing values in the measurement grid. In addition, an automatic model order determination algorithm is presented to minimize the required user interaction. Three high spatial resolution measurement examples (on a circuit board, a car door and a brake drum) have shown that the quality of the reduced measurements is much better than for the raw data (this was in particular the case when outliers were present in the measurement).

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